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# Fluctuation theory of $1/f$ noise in disordered conductors

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**Abstract.** The closed-form expression for the current noise spectral density  $S_U$  for a disordered conductor has been found starting from the general principles of Gaussian fluctuation statistics, without making *ad hoc* assumptions about the form of the distribution function of the relaxation times of the defects with internal degrees of freedom. It allows one to analyse the noise characteristics of the system consistently at arbitrary frequencies  $f$  and temperatures. On applying it to metals with frozen strains, we succeed in giving a quantitative interpretation of the phenomenon. The theory: (i) demonstrates almost  $1/f$  behaviour of  $S_U$  over a frequency interval that expands sharply with the growth of disorder in the metal; (ii) leads to 'saturation' of  $S_U$  at low  $f$ ; (iii) gives a convergent integrated intensity for the current noise; (iv) describes completely the temperature dependence of the empirical Hooge factor. Our theory also predicts a narrowing of the  $1/f$  region (to a point where it actually disappears) with temperature increase that is well known experimentally for semiconductors but has not received a consistent theoretical interpretation up to now.

## 1. Introduction

The phenomenon known as  $1/f$  (flicker, excess) noise has a long history, and a large number of attempts have been made to model it (see, for example, the reviews [1–3]). Flicker noise appears in conductors of different nature—metals, semiconductors, electrolytes—but a consistent theory for this phenomenon has not been formulated up to now. On the empirical level, the noise spectral density  $S_U$ , which is determined by voltage fluctuations against the background of the mean voltage  $\langle U \rangle$  applied to the disordered conductor, is described by the approximation formula [1–4]

$$\frac{S_U}{\langle U \rangle^2} = \frac{\alpha_H}{Nf} \quad (1)$$

where  $N$  is the number of charge carriers in the sample and the dimensionless quantity  $\alpha_H \sim 10^{-2}$ – $10^{-3}$  is the so-called Hooge factor. Questions concerning its numerical value and probable dependence on temperature and on the characteristics of the material (as well as on the frequency  $f$ ) have been subjects of theoretical discussion for a long time, but exhaustive answers to these questions remain absent from the literature.

In the present paper considering disordered metal, as an example, a consistent theory that allows one to describe the principal peculiarities of  $1/f$  noise is proposed. Working within the framework of the theory, we introduce a formalism according to which the flicker noise can be connected with random changing of electron mobility due to the fluctuation dynamics of atomic defects in the metal. Such a concept has been invoked many times to construct  $1/f$  noise models (see, for example, [3, 7–10]). However, there still have remained open some important questions concerning the consistent derivation of the  $1/f$

law, the frequency range over which it is applicable, the convergence of the  $1/f$  noise integrated intensity etc.

We will calculate the resulting noise characteristics for a disordered metal by averaging their local values over Gaussian fluctuations of an appropriate random variable. The feasibility of carrying out of such averaging in the analysis of the problem of the non-Debye relaxation of glasslike ionic conductors has been successfully demonstrated in our recent papers [5, 6]. Below, we will obtain an expression for  $S_U$  which is not only free of any *ad hoc* quantities, but also is such that, from its structure alone, we will be able to establish the physical meaning and numerical values of parameters of the type of  $\alpha_H$  which appear in empirical relations. As a direct application of our theory, we will give a detailed quantitative interpretation of the experimentally observed temperature dependencies of  $S_U$  in the  $1/f$  region for typical metals. Therefore, bearing in mind that procedures for establishing such experimental dependencies on an empirical basis date back to the work of Dutta and Horn [1], one can consider our approach as uncovering the basic physics in the Gauss-like distribution function of the relaxation times of the noise-producing defects used in [1]. The proposed theory, in principle, yields an extension to the cases of non-metallic materials—semiconductors, ionic conductors—as well (these issues are touched on in our discussion).

## 2. The fluctuation dynamics of defects and the noise spectral density

Before we turn to the mathematical formulation of the theory, wherein the electron scattering by relaxing defects is just considered as an origin of  $1/f$  noise, let us discuss the nature of such defects. The polycrystalline Ag, Cu, Au etc films for which the  $1/f$  noise phenomenon is investigated usually contain a large number of non-equilibrium defects, for example vacancies and interstitials [11] (though even nominally pure metal samples may contain a certain level of impurity, for example oxygen atoms [12, 3], with typical concentrations  $\sim 10^{-4}$  [13]). It is generally acknowledged, however, that neither their translational diffusion nor processes of the type of intrinsic defect creation/annihilation (or electron trapping) can explain the peculiarities of the current noise in the metals investigated (this was accorded special attention in the review [3]). On the other hand, it is known that in, for example, fcc Ag, the preferred shape of the intrinsic interstitial defects is a dumb-bell one [14]. Such defects break the initial cubic symmetry, and following the terminology of [3] we shall call them ‘symmetry breakers’ (SBs).

Possessing rotational degrees of freedom, the SBs can effect orientational transitions between states that are degenerate in energy, which will be manifested as a thermally activated rotational diffusion of the interstitial (the case of an impurity SB was considered in [9]; see also [2, 3]). During the scattering by SBs that are reorienting via fluctuations, the electrons of a metal will undergo random mobility changes that will just lead to a current noise spectrum.

We shall carry out the mathematical derivation of the explicit expression for the  $1/f$  noise spectral density on the basis of the general expression

$$\frac{S_U}{\langle U \rangle^2} = \frac{4}{\langle U \rangle^2} \int_0^\infty dt \cos(\omega t) \langle \delta U(t) \delta U(0) \rangle. \quad (2)$$

The temporal correlation function under the integral is to be calculated by averaging (denoted by the angle brackets) of the bilinear combination of the voltage fluctuations over the probabilities of realization of certain random characteristics of the disordered metal. Taking into account the fact that the real metal samples used in  $1/f$  noise experiments are usually

polycrystalline, a relevant characteristic, of central importance, is apparent, namely the static field of random strains frozen in a polycrystalline sample during the process of its preparation. Also, in equation (2) an averaging over a chaotic distribution of frozen SBs is also assumed to be trivial and to remain appropriate to the absence of such random fields.

In our derivation, we restrict consideration to the case that is the most widespread from the experimental point of view, in which the voltage fluctuations are measured at the same contacts that serve to provide the background current supply to the metal sample. To be specific, let us take the sample in the shape of a parallelepiped whose  $x$ -axis lies in the direction of the average (background) current  $\langle I \rangle = \langle U \rangle S / (L_x \langle \rho_{xx} \rangle)$ , where  $L_x$  is the sample length,  $S$  is the sample cross-sectional area, and  $\langle \rho_{xx} \rangle$  is the average value of the  $xx$ -component of the resistivity tensor  $\rho_{ij}$  of the metal. The voltage fluctuations in equation (2) are also proportional to  $\langle I \rangle$ , and can be represented as an integral along the sample length over the local fluctuations of the corresponding component of the resistivity tensor. Taking into account the fact that the angle brackets in equation (2) assume averaging over the sample cross-section as well, we arrive at the following expression:

$$\frac{S_U}{\langle U \rangle^2} = \frac{4}{\langle \rho_{xx} \rangle^2 V^2} \int_0^\infty dt \cos(\omega t) \int d\mathbf{r} \int d\mathbf{r}' \langle \delta \rho_{xx}(\mathbf{r}, t) \delta \rho_{xx}(\mathbf{r}', 0) \rangle \quad (3)$$

where  $V = SL_x$  is the system volume (the leading role of just resistance fluctuations in the origin of the 1/f noise has been pointed out, for example, in [15]). Before we proceed to the concrete calculation of the correlation in the presence of the above-mentioned static random strains appearing in equation (3), note that if one does not take into account the effects of the latter on SBs, then this calculation becomes trivial, because, reducing to the problem of solving a relaxation equation for orientation transitions, it will have as a solution the ordinary decay exponent with a single relaxation time. Thus the spatial correlation reduces to a  $\delta$ -function and  $S_U$  reduces to the ordinary Lorentz frequency spectrum.

Another approach to the calculation of  $S_U$  is via the specification of the resistivity fluctuation mechanism. Note, first of all, that for temperatures  $T > 100$  K (this temperature region is usually the most significant for analysis of the data on 1/f noise measurement for Ag, Cu etc films [1–3, 15]) the contribution of the processes of electron scattering by phonons (the phonon component,  $\rho_{xx}^{(\text{ph})}$ ) and by SBs (the so-called residual component,  $\rho_{xx}^{(\text{SB})}$ ) to  $\rho_{xx}$  exceeds the contribution of the electron–electron scattering, and so we shall from now on neglect the latter.

As is well known (see, for example, [16]),  $\rho_{xx}^{(\text{ph})}$  has different power dependencies on  $T$  in the regions above and below the Debye temperature  $\Theta$  (for silver,  $\Theta = 210$  K [17]). An expression which correctly reproduces the known [16] asymptotic dependencies for  $T \ll \Theta$  and  $T \gg \Theta$  for the fcc metals discussed here can be written in the form

$$\rho_{xx}^{(\text{ph})} = \frac{m^* T}{\hbar e^2 n_e} \left[ 1 + \left( \frac{\Theta}{T} \right)^4 \right]^{-1} \quad (4)$$

where  $m^*$  is the conduction electron effective mass,  $e$  is its charge, and  $n_e$  is the electron number density. We will use equation (4) to interpolate to the whole temperature range ( $\sim 100$ – $600$  K) over which experiments on 1/f noise are usually carried out.

The residual component is, in principle, formed by electron scattering by all possible types of defect, both intrinsic and extrinsic. Taking into account, for simplicity, just the electron scattering by the SBs discussed here, for the residual component of the local

resistivity tensor at the time  $t$  one may use the expression

$$\rho_{ij}^{(\text{SB})}(\mathbf{r}, t) = \frac{p_F}{e^2 n_e} \left[ \sigma n_D(\mathbf{r}) \delta_{ij} + 3\sigma' \sum_{\alpha=1}^g l_i^{(\alpha)} l_j^{(\alpha)} n_D^{(\alpha)}(\mathbf{r}, t) \right]. \quad (5)$$

Here  $p_F$  is the Fermi momentum of the metal,  $n_D^{(\alpha)}(\mathbf{r}, t)$  is an instantaneous local value of the number density of the SBs with the  $\alpha$ -orientation, and the superscript  $\alpha$  enumerates the  $g$  possible energetically equivalent orientational (as given by the unit vectors  $l^{(\alpha)}$ ) states of SBs. In the specific case of SBs in the form of dumb-bell-like intrinsic interstitials, the most stable configurations are  $g = 3$  orientations along the principal axes of the fcc lattice [14].

We will consider the total local density  $n_D(\mathbf{r})$  of randomly placed SBs to be independent of time and determined only by the sample preparation conditions, so that the following conservation law is fulfilled:

$$\sum_{\alpha=1}^g n_D^{(\alpha)}(\mathbf{r}, t) = n_D(\mathbf{r}). \quad (6)$$

The fact that equation (5) contains, in addition to the transport cross-section  $\sigma > 0$ , also the quantity  $\sigma' > -\sigma/3$  (this inequality is obtained from the condition of essential positiveness of the quadratic form which determines the energy dissipation during conduction) is a reflection of the anisotropy of the electron scattering by SBs. On carrying out an averaging in equation (5) over the orientations of the vectors  $l^{(\alpha)}$  and over spatial coordinates, we obtain for the term  $\langle \rho_{xx}^{(\text{SB})} \rangle$  in equation (3) the following expression:

$$\langle \rho_{xx}^{(\text{SB})} \rangle = \frac{p_F}{e^2 n_e} (\sigma + \sigma') n_D \quad (7)$$

where  $n_D = \langle n_D(\mathbf{r}) \rangle$  is the averaged SB density. Note that in the case where  $\sigma' = 0$  we recover the usual expression for the residual resistivity caused by electron scattering by isotropic point defects (see, for example, [16]).

The local time-dependent component  $\rho_{xx}(\mathbf{r}, t)$  of the resulting resistivity is obtained by summation of the local  $xx$ -components of the residual and phonon contributions. Before we compare the fluctuation components of these contributions, note that when low-frequency noise in metals is considered, one can certainly neglect the influence of the density fluctuations of the electron ('light') subsystem on  $\rho_{xx}(\mathbf{r}, t)$ . This is because the quantity  $n_e$  relaxes quickly, in the Maxwell time  $\sim \rho/(4\pi) \sim 10^{-19}$  s, for the resistivity  $\rho \sim 10^{-6} \Omega \text{ cm}$  [12] which is typical for metals.

In other words, from the expression  $\rho = 1/(n_e e^2 b)$ , which contains the electron mobility  $b$  (see, for example, [16]), it follows that just the fluctuations of  $b$ —which in their turn are connected to the slow reorientations of 'heavy' SBs—will determine the current noise in metals (this fact has been mentioned many times in the literature; see, for example, the review [3]). In such reorientations, the local Debye temperature determined by the metal's local shear modulus  $\mu(\mathbf{r}, t)$  [18] in the presence of SBs should, in principle, also be considered as a local dynamic quantity, relaxing together with the fluctuations of  $n_D^{(\alpha)}(\mathbf{r}, t)$ .

We will now show that, due to such fluctuations of the shear modulus, the mean square fluctuation  $\Delta\rho^{(\text{ph})}$  of the phonon contribution to the metal resistivity is numerically small in comparison with the mean square fluctuation  $\Delta\rho^{(\text{SB})}$  of the residual component. To do this, let us first determine, with the help of equation (5), the local fluctuations of the residual resistivity:

$$\delta\rho_{xx}^{(\text{SB})}(\mathbf{r}, t) = \frac{3p_F}{e^2 n_e} \sigma' \sum_{\alpha=1}^g [l_x^{(\alpha)}]^2 \delta n_D^{(\alpha)}(\mathbf{r}, t) \quad (8)$$

where we take into account the equality

$$\sum_{\alpha=1}^g \delta n_D^{(\alpha)}(\mathbf{r}, t) = 0 \quad (9)$$

which stems from the conservation law (6) for fluctuations

$$\delta n_D^{(\alpha)}(\mathbf{r}, t) = n_D^{(\alpha)}(\mathbf{r}, t) - n_D(\mathbf{r})/g$$

because SBs are assumed to perform only reorientation movements. On the other hand, the fluctuation component of the phonon contribution determined from equation (4) will, according to the above, have the same fluctuation multiplier as appears in the rhs of equation (8). Then, taking into account the dimensional structure of equation (4), we obtain the simplest estimate for the ratio of the above-mentioned mean square fluctuations:

$$\frac{|\Delta\rho^{(\text{ph})}|}{|\Delta\rho^{(\text{SB})}|} \approx \frac{4(\Theta/T)^4 m^* T}{[1 + (\Theta/T)^4]^2 \hbar \bar{\sigma} p_F} \left| \frac{\partial \ln \mu}{\partial n_D} \right| \quad (10)$$

where the derivative characterizes the relative change of the shear modulus with the defect number density, and  $\bar{\sigma}$  is the averaged cross-section of electron scattering by SBs. The value of the derivative appearing in equation (10) in the case of small concentration  $c_D = n_D/n_a \sim 10^{-4}$  in which we are interested can be extracted from the experimental data (see, for example, [13]). Indeed, in the linear approximation for  $c_D$ , the shear modulus  $\mu$  of a metal with defects can be written in the form  $\mu = \mu_0 + \mu' c_D$  where  $\mu_0$  is the shear modulus of a perfect metal, and the constant  $\mu'$  for, for example, a metal with vacancies is  $\mu' \approx -\mu_0$  [13]. Then, putting in equation (10) the typical values  $\Theta \approx 200$  K,  $m^* \approx 10^{-27}$  g,  $\bar{\sigma} \sim \sigma \approx 6 \times 10^{-16}$  cm<sup>2</sup>, and  $p_F \approx 10^{-19}$  g cm s<sup>-1</sup>, and taking as an estimate  $|\partial \ln \mu / \partial n_D| \approx 1/n_a$ , where  $n_a \approx 6 \times 10^{22}$  cm<sup>-3</sup> is the metal-atom number density, we get  $|\Delta\rho^{(\text{ph})}/\Delta\rho^{(\text{SB})}| \sim 10^{-2}-10^{-3}$  for the whole temperature range of interest to us ( $\sim 100-600$  K).

In order to find the residual resistivity correlation function which appears in equation (3), we need equations which describe the temporal evolution of the local populations of SB orientation states. As the local kinetic equation for population fluctuations, we shall use the following:

$$\frac{\partial \delta n_D^{(\alpha)}(\mathbf{r}, t)}{\partial t} = -\frac{g-1}{\tau} \delta n_D^{(\alpha)}(\mathbf{r}, t) + \frac{1}{\tau} \sum_{\beta \neq \alpha}^g \delta n_D^{(\beta)}(\mathbf{r}, t). \quad (11)$$

Introducing the relaxation time  $\tau_r = \tau/g$  and taking into account equation (6), we get from equation (11) the simple relaxation equation

$$\frac{\partial \delta n_D^{(\alpha)}(\mathbf{r}, t)}{\partial t} = -\frac{1}{\tau_r} \delta n_D^{(\alpha)}(\mathbf{r}, t). \quad (12)$$

Its solution has the form

$$\delta n_D^{(\alpha)}(\mathbf{r}, t) = \delta n_D^{(\alpha)}(\mathbf{r}) e^{-t/\tau_r} \quad (13)$$

where, for simplicity, the zero-time argument in the amplitude is omitted. The temporal correlation function of interest to us is obtained by averaging the product of equation (13) and  $\delta n_D^{(\alpha)}(\mathbf{r}')$ :

$$\langle \delta n_D^{(\alpha)}(\mathbf{r}, t) \delta n_D^{(\alpha)}(\mathbf{r}') \rangle = \langle \delta n_D^{(\alpha)}(\mathbf{r}) \delta n_D^{(\alpha)}(\mathbf{r}') e^{-t/\tau_r} \rangle. \quad (14)$$

During the calculation of this average, consider first that it is usually accepted [17] that the fluctuations appearing in equation (14) can be taken as  $\delta$ -correlated in space; then, for arbitrary  $\alpha$ , we find

$$\langle \delta n_D^{(\alpha)}(\mathbf{r}) \delta n_D^{(\alpha)}(\mathbf{r}') \rangle_P = n_a \frac{c_D}{g} \left( 1 - \frac{1}{g} \right) \delta(\mathbf{r} - \mathbf{r}') \quad (15)$$

where the normalizing factor carries the information about the average concentration  $c_D = n_D/n_a$  of defects and their degeneracy  $g$ , and by labelling the angle brackets with the subscript 'P', we are drawing attention to the Poisson character of this averaging.

As a result, the average in equation (14) which will finally determine the spectral density of the current noise in equation (3) can be presented as the product of equation (15) and  $\langle e^{-t/\tau_r} \rangle_{\text{fr}}$ , where the subscript 'fr' just indicates that the averaging is over the above-mentioned frozen random strains, the information on which is contained in  $\tau_r$  and which are of central importance for the  $1/f$  noise phenomenon. To calculate the coefficient of the Fourier transform of  $\langle e^{-t/\tau_r} \rangle_{\text{fr}}$  let us examine the concrete example of the situation with the SB structure considered for the above-mentioned case of intrinsic interstitials in the form of dumb-bell configurations orientated in the direction [100] of the fcc lattice with  $g = 3$ . Also, taking into account the polycrystalline nature of the samples used in noise experiments (see above), we should average the combinations appearing in the substitution of equation (8) into equation (3) over the crystallite random orientations as well, i.e. over all possible orientations of the orthogonal frame  $l^{(1)}, l^{(2)}, l^{(3)}$ . Carrying out the latter averaging, and calculating, with the help of equations (9) and (15), the cross-correlation ( $\beta \neq \alpha$ ) functions

$$\langle \delta n_D^{(\alpha)}(\mathbf{r}) \delta n_D^{(\beta)}(\mathbf{r}') \rangle_P = -\frac{1}{2} \langle \delta n_D^{(\alpha)}(\mathbf{r}) \delta n_D^{(\alpha)}(\mathbf{r}') \rangle_P \quad (16)$$

we get

$$\langle \delta \rho_{xx}^{(\text{SB})}(\mathbf{r}, t) \delta \rho_{xx}^{(\text{SB})}(\mathbf{r}') \rangle = \frac{18}{5} \left( \frac{p_F}{e^2 n_e} \right)^2 \sigma'^2 \langle \delta n_D^{(\alpha)}(\mathbf{r}, t) \delta n_D^{(\alpha)}(\mathbf{r}') \rangle. \quad (17)$$

Substituting equation (17) into equation (3), taking into account equations (14) and (15) and the fact that  $\langle e^{-t/\tau_r} \rangle_{\text{fr}}$ , naturally, does not depend on the spatial coordinates, and expressing  $\langle \rho_{xx} \rangle = \langle \rho_{xx}^{(\text{ph})} \rangle + \langle \rho_{xx}^{(\text{SB})} \rangle$  through equations (4) and (7) for  $g = 3$ , we arrive at the formula

$$\frac{S_U}{\langle U \rangle^2} N = A(c_D, T) \int_0^\infty dt \cos(\omega t) \langle e^{-t/\tau_r} \rangle_{\text{fr}}. \quad (18)$$

Here  $N = n_a V$  is the total number of primary cells in the sample (in the case considered, that of a simple metal,  $N$  coincides with the number of conduction electrons in the sample), and we have introduced the notation

$$A(c_D, T) = \frac{16}{5} \left( \frac{\sigma'}{\sigma} \right)^2 c_D \left[ c_D \left( 1 + \frac{\sigma'}{\sigma} \right) + \frac{T}{T_0} \frac{1}{(\Theta/T)^4 + 1} \right]^{-2}$$

with  $T_0 = \hbar n_a p_F \sigma / m^*$ . Putting in the values for silver,  $n_a = 5.8 \times 10^{22} \text{ cm}^{-3}$  and  $p_F = 1.2 \times 10^{-19} \text{ g cm s}^{-1}$ , and taking  $\sigma = 6 \times 10^{-16} \text{ cm}^2$ , we get an estimate of the range of possible values of  $T_0$  of approximately 35 000–25 000 K for the effective mass  $m^*$  in the range 1–1.5 free-electron masses.

### 3. Calculation of the fluctuation averages and comparison with experiment

With the help of the expression (18) derived above, one can analyse the current noise in disordered metals, if one calculates the average in equation (18). This average should be

calculated taking into account the fact that the local relaxation time is generally accepted [1–3] to have has the activation form

$$\tau_r = \tau_0 e^{(E_0 + \delta E)/T}. \tag{19}$$

The local activation energy—the height of the barrier which separates the energetically equivalent states of the SBs—appearing here contains, first of all, the ‘initial’ quantity  $E_0$ . For polycrystalline metals this quantity, generally speaking, does not coincide with that for an isolated SB in a perfect lattice, and contains a part connected with the existence of uniform deformations which determine the so-called ‘mean lattice’ (regarding the latter, see, for example, [19]). The random component  $\delta E$ , then, in the case of a metal, where the Coulomb fields are strongly screened, is caused, in fact, by non-uniform strain fields frozen in the polycrystal at the preparation temperature  $T_p$  (naturally, fluctuations of the pre-exponential factor  $\tau_0$  can be neglected compared to fluctuations of the exponent). These fields are produced both by extended defects—dislocations, grain boundaries—and, in principle, by point ones. The concentration of the latter, however, is rather low in typical samples ( $c_D \sim 10^{-4}$  [12]), so one can neglect direct interaction of noise-producing SBs, and consider that each of them is effectively in a frozen field of random strains. Note that the idea that the magnitude of the frozen fluctuations is determined by the preparation temperature for the metal sample was stated in an explicit form by Egami and Srolovitz [20] (see also [21]).

Let us now show that if Gaussian disorder frozen at  $T_p$  is present in the system, then, independently of the specific nature of the mechanism of the influence of the disorder on the kinetic characteristics of the conducting electrons, the temperature dependence of the resulting fluctuation term, additional to  $E_0$ , can be calculated in a general form. Indeed, denoting the random variable as  $\Omega$  (below, during the concrete example calculation, we shall consider the fluctuation part of the local relative variation of the volume due to random—non-uniform—deformations of the lattice with respect to the ‘mean lattice’ as  $\Omega$ ), and representing the fluctuation in the form  $\delta E = (\partial E_0 / \partial \Omega)_{ml} \Omega$ , where the subscript ‘ml’ indicates that the derivative is taken for the ‘mean-lattice’ state [19], we have for the average

$$\langle e^{\delta E/T} \rangle = \int \mathcal{D}\Omega \exp[(\partial E_0 / \partial \Omega)_{ml} \Omega / T] \exp[-\mathcal{F}_G\{\Omega\} / T_p] \bigg/ \int \mathcal{D}\Omega \exp[-\mathcal{F}_G\{\Omega\} / T_p]. \tag{20}$$

The normalized functional integral appearing in equation (20) is easily calculated with the Gaussian distribution function taken at  $T_p$  and expressed via the functional of the free energy of the fluctuations  $\mathcal{F}_G\{\Omega\}$  which is quadratic in  $\Omega$  (concrete examples of such calculations are given in [5, 6]; see also [17]). Taking into account the fact that  $\Omega$  is a random function of the coordinates  $\mathbf{r}$ , introducing the Fourier representation of  $\Omega$ , and calculating the Gaussian integrals that appear, we obtain

$$\langle \tau_r \rangle = \tau_{r0} \exp\left(\frac{T_*^2}{T^2}\right) \quad \tau_{r0} = \tau_0 \exp\left(\frac{E_0}{T}\right). \tag{21}$$

The characteristic temperature introduced here

$$T_* = \frac{1}{2} \left| \left( \frac{\partial E_0}{\partial \Omega} \right)_{ml} \right| \frac{\sqrt{T_p}}{(2\pi)^{3/2}} \left[ \int d\mathbf{k} A^{-1}(\mathbf{k}) \right]^{1/2} \tag{22}$$

contains the integral of the reciprocal values of the Fourier coefficients  $A^{-1}(\mathbf{k}) \geq 0$  of the Gaussian functional  $\mathcal{F}_G\{\Omega\}$ . We emphasize once again that the appearance of the additional



term  $\sim T^{-2}$  in the exponent of  $\langle \tau_r \rangle$  is a general result, and results simply from the Gaussian form of the frozen fluctuations of the random quantity  $\Omega$ , independently of its specific nature.

One can obtain a good estimate of  $T_*$  for real polycrystalline metal without using the general equation (22), i.e. avoiding the stage of direct calculation of  $A(\mathbf{k})$ . To do this, let us neglect, as was discussed above, the direct mutual influence of the SBs, and accord to the quantity  $\Omega$  the meaning of the fluctuation part of the random relative variation of the primary cell volume containing the chosen SB. Taking, like in [21, 20], the primary cell volume  $v_a$  as a fluctuation volume, and writing down the elastic fluctuation energy as  $K v_a \Omega^2 / 2$  (see, for example, [19]), where  $K$  is the elasticity effective modulus of the metal, by direct calculation of the integrals in equation (20) we arrive at equation (21), with

$$T_* = \left| \left( \frac{\partial E_0}{\partial \Omega} \right)_{\text{ml}} \right| \sqrt{\frac{T_p}{2K v_a}}. \quad (23)$$

Substituting in, here, the characteristic parameters for silver from [22]:  $T_p \approx 1200$  K (the solidification temperature) and  $K \approx 5 \times 10^{10}$  N m $^{-2}$ , and taking  $v_a \approx 20$  Å $^3$ , and also assuming that  $|\partial E_0 / \partial \Omega|_{\text{ml}} \sim E_0 \approx 1$  eV [1, 2], we obtain the estimate  $T_* \approx 1100$  K.

We return to equation (18), and use the representation

$$\frac{S_U}{\langle U \rangle^2} N = A(c_D, T) \text{Re} \int_0^\infty dt e^{i\omega t} \sum_{m=0}^\infty \frac{(-1)^m}{m!} t^m \langle \tau_r^{-m} \rangle_{\text{fr}}. \quad (24)$$

We note that the calculation of  $\langle \tau_r^{-m} \rangle_{\text{fr}}$ , in analogy with equation (21), leads to the following result [5, 6]:

$$\langle \tau_r^{-m} \rangle_{\text{fr}} = \tau_{r0}^{-m} \exp\left(m^2 \frac{T_*^2}{T^2}\right). \quad (25)$$

Consequently, the summation of the series in equation (24) can be carried out exactly and we get [5, 6]

$$\left\langle \exp\left(-\frac{t}{\tau_r}\right) \right\rangle_{\text{fr}} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty du \exp\left[-u^2 - \frac{t}{\tau_{r0}} \exp\left(2\frac{T_*}{T}u\right)\right]. \quad (26)$$

Now substituting equation (26) into equation (18) and integrating over  $t$ , we finally find

$$\begin{aligned} \frac{S_U}{\langle U \rangle^2} N &= A(c_D, T) \text{Re}\langle Z(\omega) \rangle_{\text{fr}} \\ \langle Z(\omega) \rangle_{\text{fr}} &= \frac{\tau_{r0}}{\sqrt{\pi}} \int_{-\infty}^\infty du \frac{\exp(-u^2)}{-i\omega\tau_{r0} + \exp(2T_*u/T)} \end{aligned} \quad (27)$$

where  $\langle Z(\omega) \rangle_{\text{fr}}$  (compare with [5, 6]) plays the role of a generalized impedance of the disordered metal, and is the Fourier transform of the relaxation function of the electrons with respect to their scattering by the SBs.

Expression (27) is the central result of this paper, and we shall show that  $S_U$  is  $\sim 1/\omega$  over a frequency range that sharply expands with the growth of the degree of disorder of the conductor. The quantity  $2T_*/T$  is a measure of the disorder, and we consider the behaviour of  $\langle Z(\omega) \rangle_{\text{fr}}$  in the limit where  $2T_*/T \gg 1$  which, taking into account the estimate given above for  $T_*$ , reflects a typical experimental situation (for example, according to [1],  $T < 650$  K). Then  $\text{Re}\langle Z(\omega) \rangle_{\text{fr}}$  can be calculated by the saddle-point method. The saddle point  $u_s$  of the corresponding integral is determined by the equation

$$u_s = \frac{T}{2T_*} \left[ \ln(2\pi\tau_{r0}f) - \frac{1}{2} \ln\left(\frac{1 + u_s T/T_*}{1 - u_s T/T_*}\right) \right]$$

where the frequency  $f = \omega/(2\pi)$  has been introduced. For the limit where  $2T_*/T \gg 1$  of interest to us, for which the resulting quantity  $u_s T/T_* \ll 1$ , the calculation of the saddle-point integral according to a common routine leads to the formula

$$\frac{S_U N}{\langle U \rangle^2} = \frac{A(c_D, T)T}{4\sqrt{2\pi}T_* f} \exp \left\{ -\frac{T^2}{4T_*^2} \left[ \ln(2\pi\tau_0 f) + \frac{E_0}{T} \right]^2 \right\}. \quad (28)$$

The formula obtained, equation (28) exhibits nearly  $1/f$  behaviour over the frequency interval

$$f_-(T) \ll f \ll f_+(T) \quad (29a)$$

where

$$f_{\pm}(T) = \frac{1}{2\pi\tau_0} \exp \left( -\frac{E_0}{T} \pm \frac{2T_*^2}{T^2} \right). \quad (29b)$$

As regards the estimate for  $f_+(T)$ , one should note the following remark. This estimate is, by no means, correct for temperatures above a certain level, for which the values of  $f_+(T)$  given by equation (29) do not exceed the natural boundary frequency  $\sim \tau_0^{-1}$  which is determined by the limit frequency of the atomic vibrations of the solid. Formally, expression (29) leads to a violation of this natural restriction at low temperatures, since in the derivation of equation (27) we did not put any limits on the relaxation times of the fluctuations. The point is that this restriction plays almost no role in the analysis of the low-frequency fluctuation dynamics of disordered conductors considered here; that is, it does not have any effect on the form of equation (28) or on that of  $f_-(T)$ . The correction to the estimate for  $f_+(T)$  given as (29) can be realized on the qualitative level by changing the lower limit of the integral in equation (27) by  $-E_0/(2T_*)$  (compare with the analogous model forms in [21, 22]), to ensure that at low temperatures the natural condition  $f_+(T) \sim \tau_0^{-1}$  is obeyed.

Estimation of the lower boundary in inequality (29) for  $T = 300$  K for the case of a metal with the typical parameter values  $\tau_0 = 10^{-14}$  s,  $E_0 = 1$  eV [1, 2], and  $T_* = 1300$  K, leads to a practically unmeasurable value,  $f_-(300\text{K}) = 1.3 \times 10^{-20}$  Hz; for  $T = 500$  K, we obtain  $f_-(500\text{K}) = 1.8 \times 10^{-3}$  Hz. For the upper boundary, we get  $f_+(300\text{K}) = 5.2 \times 10^{12}$  Hz and  $f_+(500\text{K}) = 10^9$  Hz. For the range  $f \ll f_-(T)$ , as is seen from equation (27),  $S_U N / \langle U \rangle^2$  goes to the ‘saturation’ value

$$A(c_D, T)\tau_0 \exp(E_0/T + T_*^2/T^2).$$

As for the region  $f \gg f_+(T)$  at ‘high’ temperatures, for which  $f_+(T)$  is determined by the expression (29), the regime where

$$S_U N / \langle U \rangle^2 = A(c_D, T)(4\pi^{3/2})^{-1} \tau_0^{-1} [\exp(-E_0/T + 2T_*^2/T^2)] f^{-2}$$

materializes; but at ‘low’ temperatures, for which, according to the above,  $f_+(T) \sim \tau_0^{-1}$ , for the noise spectral density in the high-frequency limit we find

$$S_U N / \langle U \rangle^2 \sim A(c_D, T)\tau_0^{-1} f^{-2}.$$

It is useful to note that the tendency of the spectral density towards ‘saturation’ in the low-frequency limit predicted by our theory produces convergence of the noise integrated intensity, so the ‘paradox’ of the divergence of the mean square of the fluctuations of a quantity responsible for the  $1/f$  noise frequently discussed in the literature (see, for example, the review [2]) does not arise at all.

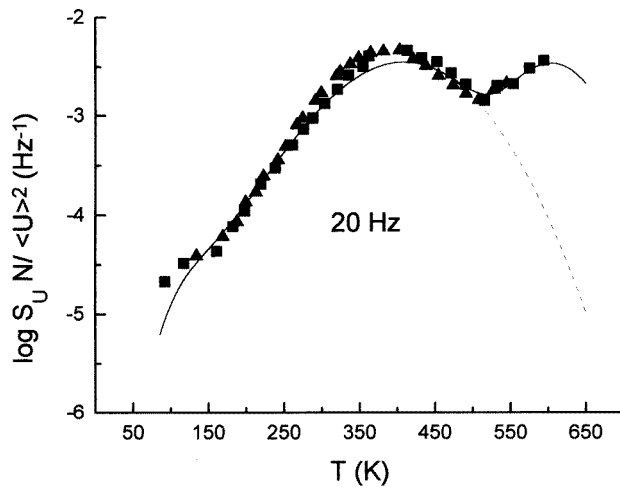
Thus, equation (27) (as well as its approximation variant (28)), being derived on general grounds, demonstrates the consistency of the derivation of the law for the current noise. Moreover, by comparison of equation (28) with expression (1), one can establish the

complete temperature and frequency dependences of the factor  $\alpha_H$ , now also containing the information about the material characteristics ( $\tau_0$ ,  $E_0$ , and quantities appearing in  $T_*$ ,  $A(c_D, T)$ ) of the disordered conductor. It is interesting to note that, in a certain sense, the quantity  $\alpha_H$  can be considered as nearly constant under the condition  $T \ll E_0/|\ln(2\pi\tau_0 f)|$ , for which the main (exponential) temperature dependence almost disappears. Choosing for  $\tau_0$ ,  $E_0$ , and  $T_*$  the values given above, and using as an estimate  $c_D = 10^{-4}$ , we find, for example, for  $f = 20$  Hz and  $T = 250$  K, the value  $\alpha_H \approx 5 \times 10^{-3}$ , which is close to the commonly accepted value of the factor  $\alpha_H$  in the empirical formula of Hooge [1–4].

It is useful to analyse to what extent our formula (28) satisfies the relation

$$-\frac{\partial \ln S_U}{\partial \ln f} = 1 - \frac{1}{\ln(2\pi\tau_0 f)} \left( \frac{\partial \ln S_U}{\partial \ln T} - 1 \right)$$

obtained by Dutta and Horn [1] under somewhat restrictive conditions. At sufficiently low temperatures one can neglect the temperature-dependent term in our  $A(c_D, T)$  (with the parameters given above, such neglect is permissible for  $T < 90$  K). In this case, as one can verify, the relation of Dutta and Horn is obeyed exactly. But for high temperatures, when just including the second term in  $A(c_D, T)$  is crucial, the Dutta and Horn relation is violated. The cases of violation of this relation have been noted earlier (see the discussion of such circumstances in, for example, [3]).



**Figure 1.** The experimental temperature dependence of  $S_U N / \langle U \rangle^2$ , for Ag for  $f = 20$  Hz (symbols, [1]), and the result given by our theory (solid line); the dashed line corresponds to the theory based on the consideration of a single type of relaxing defect. (For the parameters, see the text.)

The formulae (27) and (28) derived by us can be considered as the theoretical justification fitting the expressions used in [1] for use in numerical processing of the experimental data on  $1/f$  noise. In figure 1 the experimental (triangles and squares [1]) temperature dependencies of  $S_U N / \langle U \rangle^2$  at  $f = 20$  Hz for Ag films on different substrates are shown, together with our theoretical fitting according to equation (28) (the dashed line). The fitting procedure, being realized via processing of the experimental data from [1] up to 500 K with the help of the program Origin40, with the fixed values of the parameters  $T_0 = 25\,000$  K,  $c_D = 10^{-4}$ , and  $\tau_0 = 1 \times 10^{-14}$  s, led to the following values of the adjustable parameters:  $E_0 = 1.005 \pm 0.007$  eV,  $T_* = 1334 \pm 16$  K,  $\sigma'/\sigma = 1.892 \pm 0.005$ .

From the results obtained, it is seen that our theory allows extraction, with sufficiently high accuracy, from the experimental data the numerical values of such basic parameters of the noise-producing material as  $E_0$ ,  $T_*$ , and  $\sigma'/\sigma$ . Note that the value of  $E_0$  obtained from the experiment, as well as the chosen values  $c_D$  and  $\tau_0$ , are close to those commonly accepted for condensed media [1–3].

The observed upturn in the noise magnitude seen in figure 1 for  $T > 500$  K could be explained by the existence of SBs of at least one more kind in the film (such a possibility was discussed as far back as in [1]). The contribution of SBs of the second kind to  $S_U$  is described by just the same formula, equation (28), but, generally speaking, with other values of the activation energy and characteristic temperature. If one accepts the additivity of the contributions, and retains the values of the remaining parameters, then, having taken for the SBs of the second kind the parameters (marked by primes)  $E'_0 = 1.44$  eV and  $T'_* = 890$  K, we get the solid line displayed in figure 1. Thus our calculation can be made to agree well with the experimental data on current noise in metals.

#### 4. Discussion

In this paper we concentrated our attention on the case of flicker noise in metals. However, as far back as in Hooke's papers (see, for example, [23]) the problem of  $1/f$  noise in liquid electrolytes was discussed, and, relatively recently, there have appeared papers (see, for example, [24]) in which the measurement of current noise in solid ionic conductors was reported. Our theory can be modified to provide a description of the current noise in disordered ionic conductors and semiconductors as well.

In particular, in liquid electrolytes where such notions as SBs and the electron conduction zone lose their meaning, one should expect the  $1/f$  noise to be determined by the relaxation of ionic component fluctuations. With account taken of the Coulomb nature of such fluctuations, their dynamics, in application to the problem of  $1/f$  noise, can be described on the basis of ideas developed in our papers [5, 6] (we separately emphasize the case of ergodic disorder considered in [6]). The corresponding results, which allow us to explain the anomalously large ( $\sim 10$ ) values of the Hooke factor for electrolytes [23], will be given elsewhere later.

The case of semiconductors, in particular, is of special interest, because for them our theoretical conclusion that the noise spectral density 'saturates' as  $f \rightarrow 0$  finds clear experimental confirmation, and has long been discussed on an empirical level (see, for example, [25, 1]) without making connection with basic physics (let us mention, though, the Monte Carlo simulation of the  $1/f$  noise in doped semiconductors in [26]). Another major conclusion of our theory is that the  $1/f$  region narrows as the temperature increases when  $f_+(T) \rightarrow f_-(T)$  in a conductor with frozen disorder. However, for typical metals with  $T_* \sim 1000$  K (see above), this narrowing would be noticeable only at high temperatures. Meanwhile, for semiconductors the effect of the narrowing of the  $1/f$  noise region to a point where it disappears is well known experimentally (see, for example, [27]), but has not so far received a consistent theoretical interpretation.

Note that for semiconductors and ionic conductors the connection between their noise properties and the non-Debye character of the electric relaxation can be observed the most unambiguously; we dedicated our papers [5, 6] to the theory of this connection. Let us emphasize that the attempts undertaken earlier to link these two fundamental phenomena were, according to [3], 'untenable' (we hope to present a consistent theoretical discussion of this link later on).

Finally, of special interest is the  $1/f$  noise in whiskers (see, for example, [28, 29]),

for which its intensity can exceed by one to two orders of magnitude the noise intensity in deposited metal films of the same volume (see also [3] and the references therein). It seems that the ‘bulk’ theory developed in the present paper cannot be applied directly to the quantitative interpretation of experiments on whiskers (the first to be considered were bismuth ones [29]) for which, one may assume, the surface effects make a substantial contribution to the  $1/f$  noise [3]. That is, for semimetals of the bismuth type, it may be connected with the existence of a sufficiently large adjacent surface layer of space charge (this is because of the much lower—by approximately four orders of magnitude—concentration of free carriers in bismuth than in normal metals [2]) which may be the main source of  $1/f$  noise in such systems of lowered dimensionality. It is useful to note that in semiconductors, for which the carrier concentration is as a rule relatively small, the significant role of the surface contribution to the  $1/f$  noise is well known (see, for example, [2]). At the same time, in [3] certain doubts were expressed concerning the reality of anomalously large noise intensities in whiskers of normal metals.

## 5. Summary

In the present paper we have proposed a fluctuation theory of current noise in disordered conductors. Being based on general principles of Gaussian fluctuation statistics, the theory is free of the *ad hoc* assumptions usually made—for example, that one can choose a form for the energy distribution function of the relaxation times of the noise-producing defects in accordance with the experimental data. As a result, we succeeded in finding a closed-form expression for the noise spectral density that allows us to analyse the noise characteristics of a material consistently, for arbitrary frequencies and temperatures. In particular, the theory allows us:

- (i) to demonstrate the almost  $1/f$  behaviour of  $S_U$  over a frequency interval whose limits are explicitly expressed in terms of the degree of disorder of the material and which sharply expands with the growth of the degree of disorder;
- (ii) to achieve a convergent integrated intensity of the current noise;
- (iii) to describe completely the temperature dependence of  $S_U$  for any given frequency, and thus to confer a clear quantitative meaning of the empirical Hooge factor;
- (iv) to point out the limit value of  $S_U$  as  $f \rightarrow 0$  (and the behaviour of  $S_U$  as  $f \rightarrow \infty$ );
- (v) to give the temperature dependence of the frequency at which  $S_U$  reaches ‘saturation’, and to establish that the  $1/f$  region narrows as the temperature increases.

In an application of our theory to disordered metals with frozen random strains, we succeeded in quantitatively manipulating experimental data on  $1/f$  noise using values of parameters which were not at variance with estimates commonly accepted in condensed matter and which, in principle, constitute independent measurements.

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